A Sensitivity Figure for Yield Improvement

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Abstract —A new network sensitivity figure for use in gradient-type optimizers which accounts for random parameter variations encountered during manufacturing is presented. The difference between conventional sensitivity descriptions and the new sensitivity figure is analyzed and explained. Two examples are presented where yield improvement is obtained using the new sensitivity figure in a gradient-type optimizer.

I. Introduction

A PROBLEM IN the development of marketable microwave circuits (such as IC's) is the large number of iterations required before the final design is complete. The final design must meet all circuit criteria and, equally important, must be manufactured with a high yield using components whose values are not exactly specified. Yield is the percentage of circuits which pass all specifications during manufacture. It is common practice today to optimize a circuit design for certain performance criteria, but manufacturability (high yield production using uncertain component values) is generally considered only when the design is on the manufacturing floor. Ideally, the concept of manufacturability should appear in the initial design process, so that costly, time-consuming fabrication and testing cycles can be avoided.

Optimization is a powerful and useful tool in modern circuit design. A flow chart of the optimization process using gradient methods is shown in Fig. 1.

In general, when using gradient methods, a sensitivity analysis is performed and the results of this analysis drive a parameter-modification routine. The parameter-modification routine chooses new parameter values which are better in the sense described by the sensitivity block. In present-day optimizers sensitivity is determined by considering circuit performance only at single values of the circuit parameters. This can lead to designs with poor manufacturing yield [3]. If circuit performance over the entire range of component values encountered in manufacturing is considered by the optimizer, a more manufacturable design should result. The sensitivity analysis block is the logical place to include this information.

A new sensitivity figure for use in gradient method optimization is introduced in this paper. This sensitivity figure incorporates performance over the range of component values that the circuit will encounter during manufacture. The main hypothesis of this paper is that use of this new sensitivity figure in a gradient optimizer will result in a circuit with improved yield. This work is similar to that of Kjellerstrom *et al.* [4] in that they also pose their problem as a criteria optimization problem; however, their approach requires tuning and does not make direct use of gradient methods. This work differs from most other statistical design literature in that it proposes to optimize a performance function over the tolerance region rather than attempting to directly optimize yield. It is felt that the approach used here can lead to computational savings over other methods as it is more compatible with existing CAD systems. Monte Carlo techniques similar to [5] and [6] are employed in this paper to evaluate the sensitivity figure, although other methods [7], [8] should give additional computational benefits.

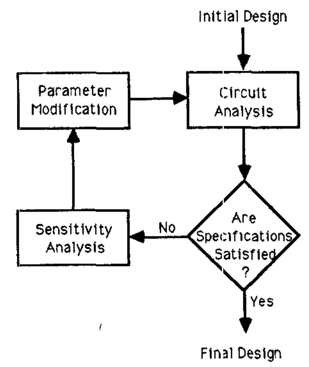

Fig. 1. The gradient optimization process.

In Section II, the standard sensitivity figure for gain is presented. The new sensitivity figure is then developed and the two are compared. Section III examines the new sensitivity figure in a cascade electrical network context. This cascade analysis lends much insight into the proposed sensitivity figure. Two examples of network optimization from a standard gradient optimizer (which uses the standard sensitivity analysis) and the new circuit optimizer (which uses the sensitivity analysis proposed here) are given in Section IV. Improved yield using the new sensitivity analysis in the gradient optimizer is shown.

II. Sensitivity Figure Description

Assume that a circuit is described by N parameters, $X = (x_1, x_2, \cdots, x_N)$. These parameters can be resistance, inductance, and capacitance values, S-parameter descriptions of active devices, dimensions of microstrip line, process variables, or any other parameters which affect the performance. Let the nominal values of the parameters be

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given by $X_0 = (x_{10}, x_{20}, \cdots, x_{N0})$. Furthermore assume, without loss of generality, that the performance criterion is gain, which is represented as $G(x_1, x_2, \cdots, x_N)$. A very common way of characterizing the gain sensitivity to component x_i is to calculate the sensitivity figure $S(G, x_i)$ given by [1] as

$$S(G, x_i) = \frac{x_i}{|G|} \frac{\partial |G|}{\partial x_i}\bigg|_{X = X_0,\ \text{the nominal values.}}$$

To examine the properties of this sensitivity figure the gain function G is expanded in a series expansion about the nominal values of the components. To simplify the notation, normalized variables are used such that the nominal value of each component equals zero ($x_{i0} = 0$) and the tolerance on each component is $-1 \leq x_i \leq 1$. All the important features of this argument are illustrated when there are only two components, and the notation is greatly simplified; therefore assume that there are only two parameters in $G = G(x_1, x_2)$. The extension to more than two parameters is straightforward, but tedious. The series expansion of the gain about the normalized nominal values is

$$G(x_1, x_2) = G_0 + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_{12} x_1 x_2 + \alpha_{11} x_1^2 + \alpha_{22} x_2^2 + \alpha_{111} x_1^3 + \alpha_{122} x_1 x_2^2 + \alpha_{112} x_1^2 x_2 + \cdots$$

The partial derivative of G with respect to x_i is included in $S(G, x_i)$. To better understand the partial derivative, its evaluation is expanded as

$$\frac{\partial G(x_1, x_2)}{\partial x_1} = 0 + \alpha_1 + 0 + \alpha_{12} x_2 + 2\alpha_{11} x_1 + 0 + 3\alpha_{111} x_1^2 + \alpha_{122} x_2^2 + 2\alpha_{112} x_1 x_2 + \cdots.$$

If the derivative is evaluated at the nominal values [1], which is usually the case, the result is

$$\frac{\partial G(x_1, x_2)}{\partial x_1}\bigg|_{X = X_0} = \alpha_1. \qquad (1)$$

As an alternative, it is proposed here that an average derivative be used as the basis for a sensitivity figure. By averaging the derivative over all the other component values, the effects of uncertainty in the component values are included.

Let $\overline{G}(x_i)$ equal the average gain with respect to all the parameters except x_i. For the two-parameter case, we have

$$\overline{G}(x_1) = \int_{-1}^{1} G(x_1, x_2) p(x_2)\, dx_2$$

where $p(x_2)$ is the probability density function on the second parameter. The derivative of the average gain is given by

$$\frac{\partial \overline{G}(x_1)}{\partial x_1} = \int_{-1}^{1} \{ \alpha_1 + \alpha_{12} x_2 + 2\alpha_{11} x_1 + 3\alpha_{111} x_1^2 + \alpha_{122} x_2^2 + 2\alpha_{112} x_1 x_2 + \cdots \} p(x_2)\, dx_2.$$

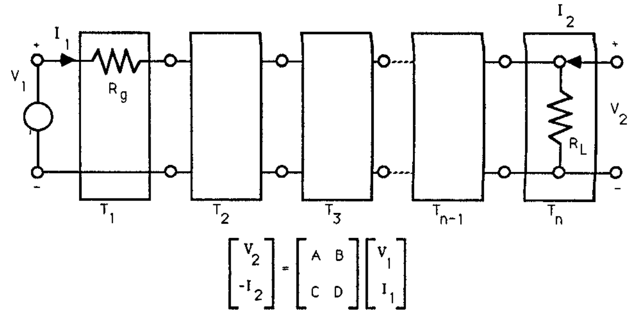

Fig. 2. Cacaded two-port.

Since

$$\int_{-1}^{1} p(x_2)\, dx_2 = 1$$

and letting the ith moment of x_2 be given by

$$\int_{-1}^{1} x_2^i p(x_2)\, dx_2 = \overline{x}_2^i$$

we obtain

$$\frac{\partial \overline{G}(x_1)}{\partial x_1} = \alpha_1 + \alpha_{12} \overline{x}_2^1 + 2\alpha_{11} x_1 + 3\alpha_{111} x_1^2 + \alpha_{122} \overline{x}_2^2 + 2\alpha_{112} x_1 \overline{x}_2^1 + \cdots.$$

Evaluating at the nominal value for x_2, we obtain

$$\frac{\partial \overline{G}(x_1)}{\partial x_1}\bigg|_{X = X_0} = \alpha_1 + \alpha_{12} \overline{x}_2^1 + \alpha_{122} \overline{x}_2^2 + \cdots. \qquad (2)$$

The higher order dependence on x_2 is maintained in (2) by simply replacing the x_2 parameter by its appropriate moment in the expansion. Thus the higher order behavior of the derivative is preserved. We call this the average derivative. This derivation shows that the derivative (1) and the average derivative (2) can be significantly different. Note that the sign of (1) does not always equal the sign of (2). Choosing either (1) or (2) in the sensitivity block of a gradient optimizer can influence the performance of the optimizer.

The next section gives a description of the sensitivity functions in the context of cascaded networks. Much insight into the properties of the average derivative can be gained from this analysis.

III. Sensitivity Analysis of Interconnected Two-Port Networks

Consider the case of the cascaded electrical subnetworks [2] shown in Fig. 2. Interconnections between subnetworks can be any of the commonly used types [2], namely series, parallel, hybrid, and cascade. The total performance of a cascade can be determined by the performance of each subnetwork using the subnetwork's *ABCD* matrix description.

The overall *ABCD* matrix for the n-subnetwork cascade in Fig. 2 is expressed as

$$T(X) = T_1 \quad T_2 \cdots T_{k-1} \quad T_k \quad T_{k+1} \cdots T_n$$

where T_k is the $ABCD$ matrix of the kth subnetwork of the cascade, $k=1,2,\cdots,n$, and X is a vector consisting of the m variable network elements, so that

$$X=\{x_1,x_2,\cdots,x_m\}.$$

In a manufacturing environment each x_i is assigned a nominal (center) value, a tolerance range of acceptable values around the nominal, and a density function $p(x_i)$. Further, X can be subdivided into parameter groups of m_k elements which are associated only with the T_kth subnetwork:

$$X_k=\{x_{s_k+1},x_{s_k+2},\cdots,x_{s_k+m_k}\}$$

where

$$s_k=\begin{cases}0, & k=1\\ \sum_{j=1}^{k-1} m_j, & 1<k\le n.\end{cases}$$

An unnormalized sensitivity figure similar to (1) is defined in terms of the $ABCD$ matrix description and the nominal values of the circuit parameters and is given as [2]

$$\frac{\partial T(x_i)}{\partial x_i}=T_1\quad T_2\cdots T_{k-1}\frac{\partial T_k}{\partial x_i}T_{k+1}\cdots T_n\big|_{x_1=\mathrm{nom},\,x_2=\mathrm{nom},\cdots,\,x_m=\mathrm{nominal}} \quad (3)$$

where only x_i lies in the kth subnetwork. It is important to note that T_k can be a function of many parameters, one of which is x_i.

In a manner similar to that of Section II, an average derivative can be defined. First, the average of T_k, $\overline{T}_k$, is examined:

$$\overline{T}_k\equiv\int\int\cdots\int T_k P(x_{s_k+1},\cdots,x_{s_k+m_k})\,dx_{s_k+1},\cdots,dx_{s_k+m_k}$$

where $P(x_{s_k+1},\cdots,x_{s_k+m_k})$ is the joint density on the parameters.

This average can be extended to include all T_k:

$$\overline{T}=\int\int\cdots\int TP(x_1,\cdots,x_m)\,dx_1,\cdots,dx_m. \quad (4)$$

An average derivative for each T_k can be defined as

$$\overline{\frac{\partial T_k}{\partial x_i}}(x_i)\equiv\int\int\cdots\int\frac{\partial T_k}{\partial x_i}P(x_{s_k+1},\cdots,x_{i-1},x_{i+1}\cdots,x_{s_k+m_k})\,dx_{s_k+1},\cdots,dx_{i-1},dx_{i+1},\cdots,dx_{s_k+m_k}. \quad (5)$$

It is important to note that (5) is a function of x_i. Finally, assuming parameter x_i lies in subnetwork T_k, and the parameters in each subnetwork are independent, the average derivative of T with respect to x_i is given as

$$\overline{\frac{\partial T}{\partial x_i}}(x_i)\equiv\int\int\cdots\int\frac{\partial T}{\partial x_i}P(x_1)\cdots P(x_{i-1})P(x_{i+1})\cdots P(x_m)\,dx_1,\cdots,dx_{i-1},dx_{i+1},\cdots,dx_m$$
$$=\overline{T}_1\overline{T}_2\cdots\overline{\frac{\partial T_k}{\partial x_i}}(x_i)\cdots\overline{T}_n. \quad (6)$$

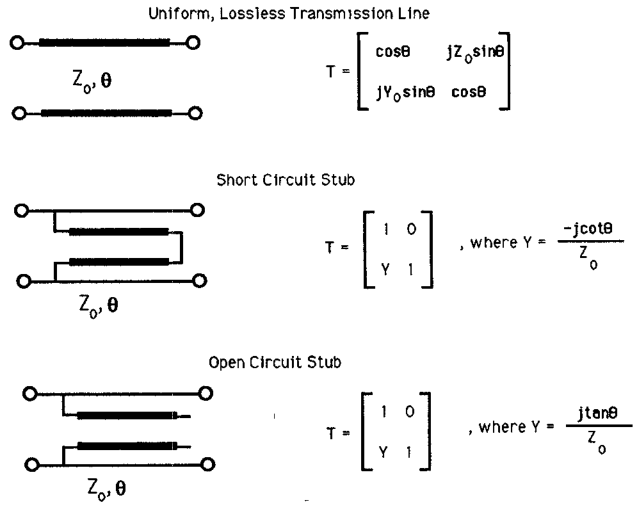

Fig. 3. $ABCD$ matrix description for common subcircuits.

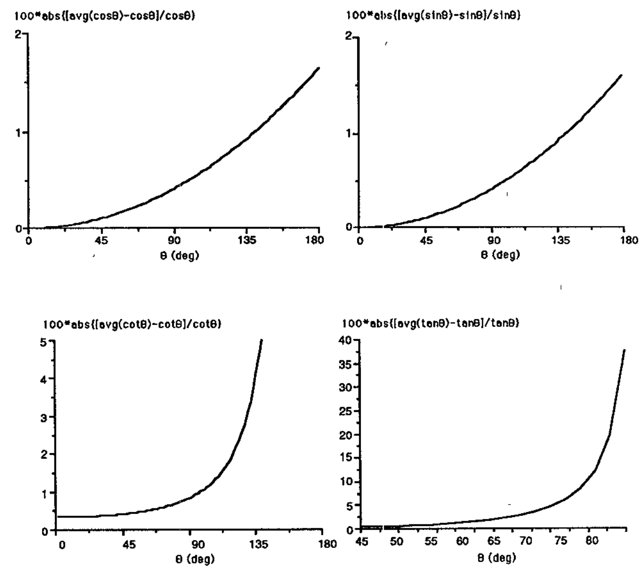

Fig. 4. A comparison of the magnitude of the normalized difference between the nominal value and the average value for the elements in Fig. 3, with ±10 percent uniform distributions on the parameters.

In general (3) and (6) can have significantly different values. To examine the differences between T_k and $\overline{T}_k$, Fig. 3 shows the $ABCD$ matrix for the open- and short-circuit stub, and the uniform, lossless transmission line. Fig. 4 shows 100 times the magnitude of the normalized difference between the average (using nominal ± 10 percent uniform distributions on the parameters) and the nominal value for the various trigonometric functions appearing in the transmission matrix description for the elements in Fig. 3. It is seen that for a wide range of θ, $T\neq\overline{T}$. For this reason differences may arise in the evaluation of the gain slope and the average gain slope for a network. Furthermore, the differences between T_k and $\overline{T}_k$ are multiplicative with the other errors, and large differences between the evaluation of the two functions can result. To demonstrate the effectiveness of using the average gain slope in the sensitivity analysis section of a gradient optimizer, two examples are presented in the next section.

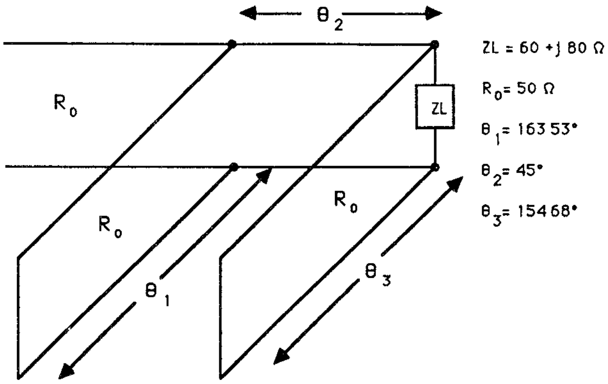

Fig. 5. Double stub tuner used in the example.

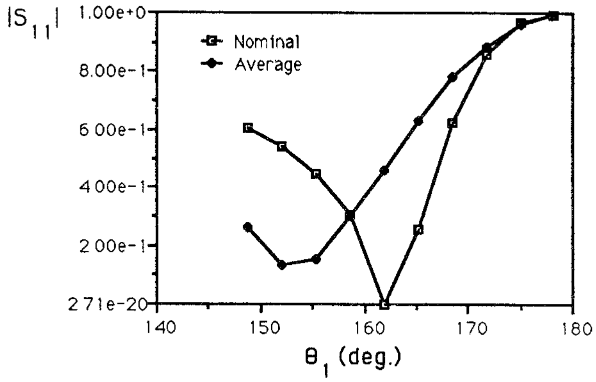

Fig. 6. Normal and average performance for the parameter θ_1.

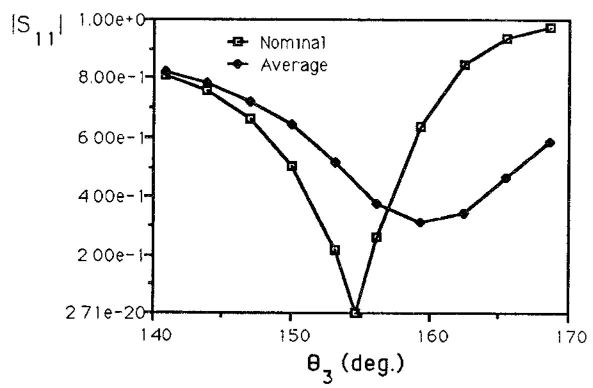

Fig. 7. Normal and average performance for the parameter θ_3.

IV. Example of Network Performance/Yield Improvement

The first example network is a simple $\lambda/8$ double stub tuner used to match a load of $60+j80$ to 50 Ω. This circuit was optimized using the nominal gradient optimizer for the magnitude $S_{11} < 0.01$ at 12 GHz. The circuit configuration and with the nominal values from the gradient optimizer are shown in Fig. 5. For this example, circuit performance is defined as $|S_{11}|$.

The average sensitivity figure operates on the average performance, while the standard sensitivity figure operates on the nominal performance. Plots of the average and nominal performance for θ_1 and θ_3 are shown in Figs. 6 and 7. Each optimizer attempts to place the optimal nominal value at the minimum of these curves. it is easy to see that each optimizer determines different optimal values.

TABLE I
Optimized Parameter Values

f = 12 GHZ VP=3E8 m/s α = 0	Characteristic Impedance R (Ω)		Line length (θn elec deg)	
Parameter # n	nominal optimizer	average optimizer	nominal optimizer	average optimizer
1	50 Ω	50 Ω	163.53°	151.20°
2	50 Ω	50 Ω	45.00°	47.50°
3	50 Ω	50 Ω	154.68°	158.40°

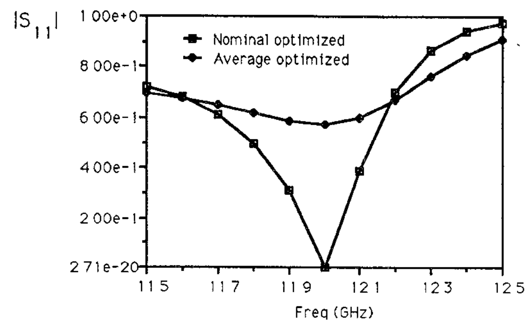

Fig. 8. Swept frequency response of the optimized circuits.

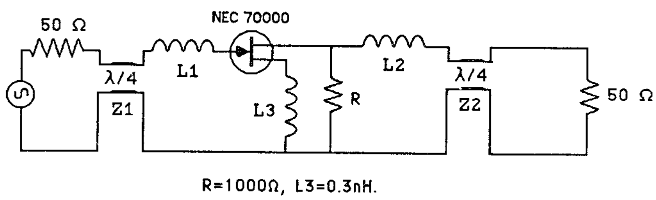

Fig. 9. The 4-GHz single-stage narrow-band amplifier.

Using the final parameter values from the nominal optimizer, the circuit was then optimized using the circuit optimizer proposed here. This is called the yield improvement design. The average performance evaluation made by the sensitivity analysis was estimated using Monte Carlo techniques as described in [3]. The nominal optimizer and the average optimizer parameter values are shown in Table I. A swept frequency response of the two designs is shown in Fig. 8.

Next, a yield analysis using Touchstone MC was made on the two designs. All line impedances and line lengths were varied uniformly ± 10 percent from nominal with a pass criterion of $|S_{11}| < 0.5$. The nominal optimized circuit showed a yield of 11.4 percent, while the average optimized circuit showed a yield of 13.9 percent.

A second example is a single-stage narrow-band 4-GHz amplifier, shown in Fig. 9. The FET used is the NEC 70000. The circuit was optimized with a nominal gradient optimizer using the criterion

$$|S_{11}| < 0.35, \quad |S_{22}| < 0.35, \quad \text{and} \quad |S_{21}| > 15.$$

The transmission line lengths, L3, and R were kept constant. The nominal values from this optimization are shown in Table II. The yield criterion as well as the performance

TABLE II
OPTIMIZED PARAMETER VALUES

	nominal optimized values	average optimized values	percent change
Z1	15.20Ω	27.00Ω	78%
L1	3.67nH	3.70nH	.82%
Z2	27.50Ω	50.00Ω	82%
L2	5.20nH	4.50nH	-14%
Yield	26%	88%	

criterion for the average optimization was

$$|S_{11}| < 0.45, \quad |S_{22}| < 0.45, \quad \text{and} \quad |S_{21}| > 5.$$

Using the above criteria and varying all components ± 10 percent as well as the FET S-parameters ± 5 percent (uniformly and independently distributed), the nominal circuit yield was 26 percent. The average optimized circuit took eight iterations to converge and the final circuit values are shown in Table II. The yield for this circuit was 88 percent. This is a good example of how a nominal optimized circuit can exhibit poor manufacturability and is similar to many of the examples we have encountered.

V. CONCLUSIONS

Although circuit optimization is a powerful and much used tool in the circuit design industry, standard gradient optimization often results in a circuit that cannot be manufactured with high yield. The authors have observed that the yield of gradient-optimized designs can be improved by choosing different nominal values for the circuit design. This has been verified with simple networks such as the ones presented in this paper, a three-transistor low-noise amplifier, and others [3]. In general, the yield of standard gradient-optimized designs can be improved by proper design centering.

In an attempt to incorporate manufacturability into the optimization process, a new network sensitivity figure (for use in gradient-type optimizers) which takes into account random parameter variations encountered in manufacturing has been presented. The difference between conventional sensitivity figures and the new sensitivity figure is due to differences between the circuit performance evaluated at the nominal component values and the circuit performance averaged over the parameter values encountered during manufacture. The examples indicate that conventional approaches for calculating component parameter sensitivities can lead to optimized designs with poor manufacturability, and that yield improvement is possible using an optimizer which incorporates this new sensitivity analysis.

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REFERENCES

[1] K. C. Gupta, R. Garg, and R. Chadha, *Computer Aided Design of Microwave Circuits: Tolerance Analysis.* Dedham, MA: Artech House, 1981.

[2] K. W. Lobst and K. A. Zaki, "An optimization technique for lumped-distributed two ports," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 2167–2171, Dec. 1982.

[3] A. MacFarland et al., "Centering and tolerancing the components of microwave amplifiers," in *Proc. 1987 MTT-S Int. Conf.*, June 1987.

[4] G. Kjellerstrom, L. Taxen, and L.-E. Blomgren, "Optimization methods for statistical network design," in *IEEE Proc. 75 ISCAS*, pp. 321–324.

[5] K. Singhal and J. F. Pinel, "Statistical design centering and tolerancing using parametric sampling," in *Proc. Int. Symp. CAS*, 1980, pp. 822–885.

[6] R. S. Soin and Robert Spence, "Statistical exploration approach to design centering," *Proc. Inst. Elec. Eng.*, vol. 127, pt. G, no. 6, pp. 260–269, Dec. 1980.

[7] S. W. Director and G. D. Hachtel, "The simplicial approximation approach to design centering," *IEEE Trans. Circuits Syst.*, vol. CAS-24, pp. 363–372, 1977.

[8] K. S. Tahim and Robert Spence, "A radial exploration approach to manufacturing yield estimation and design centering," *IEEE Trans. Circuits Syst.*, vol. CAS-26, pp. 768–774, Sept. 1979.

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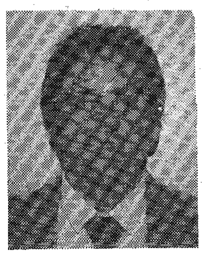

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